HYDRODYNAMICS OF DRAINING A NON-NEWTONIAN LIQUID FROM AXISYMMETRIC VESSELS

G. R. Shrager and V. A. Yakutenok UDC 532.135

The effect of rheological parameters on the time behavior of the free surface, residual mass in the vessel, and flow structure is determined for the process of draining a nonlinear-viscoplastic liquid from axisymmetric vessels.

At present there are available a large number of studies on the process of drainage. The majority of these consider nonviscous liquids [1, 2]. Draining of a highly viscous Newtonian liquid was considered in quite full detail in [3]. An estimate of residual liquid mass in the film on the walls of conical and cylindrical vessel after draining of a power-law and viscoplastic liquid was obtained in [4].

1. Formulation of the Problem. We will consider the slow escape (Re ≪ 1) of a highly viscous liquid having an initially horizontal free surface from axisymmetric vessels.

For low Reynolds numbers the flow is described by the following equations:

$$\nabla\cdot\Pi+\rho\mathbf{g}=0, \tag{1}$$

$$\nabla\cdot\mathbf{V}=0. \tag{2}$$

Upon deformation a wide class of liquids manifest complex rheological properties, including, in particular, a nonlinear dependence of viscosity upon shear velocity and the presence of a yield point. The most general model suitable for describing flow of a nonlinear viscoplastic liquid over a wide range of shear velocities is that proposed by Shul'man [5]:

$$\tau = {}_i^j\, 2\left[\frac{\tau_0^{1/n}}{A^{1/m}}+\mu^{1/m}\right]^n A^{n/m-1}\,\dot{e}_{ij},\; i,\; j=1,\,2,\,3. \tag{3}$$

From Eq. (1) with use of the expression for the stress tensor $\Pi = -pI + T$ and condition (2) we can obtain an equation for the pressure

$$\nabla^2 p=\nabla\cdot(\nabla\cdot T). \tag{4}$$

In place of original system of Eqs. (1), (2), we will use the system of Eqs. (1), (4). Equivalence of the solutions is insured by satisfaction of the continuity equation on the boundaries [6].

The solution of system (1), (4) must be found for the following boundary conditions:

$$\mathbf{V}|_{\mathbf{x}\in S_2}=\mathbf{V}_0(\mathbf{x}), \tag{5}$$

$$\mathbf{s}\cdot\Pi\cdot\mathbf{n}|_{\mathbf{x}\in S_1}=0, \tag{6}$$

$$\mathbf{n}\cdot\Pi\cdot\mathbf{n}|_{\mathbf{x}\in S_1}=-p_0, \tag{7}$$

$$\nabla\cdot\mathbf{V}|_{\mathbf{x}\in S}=0. \tag{8}$$

The motion of the free surface obeys the kinematic condition

$$\frac{\partial f}{\partial t}+\mathbf{V}\cdot\nabla f=0. \tag{9}$$

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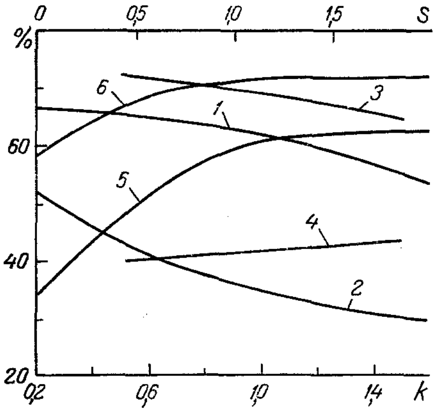

Fig. 1. Mass residue referred to mass at initial time vs nonlinearity parameter k (1, W = 0.126; 2, W = 2, conical vessel; 3, W = 0.8; 4, W = 12.8, cylindrical tank) and vs plasticity parameter S (5, W = 2, conical vessel; 6, W = 3.2, cylindrical tank).

The solution region is either a cylindrical tank or a conical vessel connected to a cylindrical drain insert. In view of the axial symmetry only half of the flow region need be considered. Symmetry conditions are satisfied on the axis of symmetry. The flow rate at the output of the drain orifice is considered known, with the velocity profile coinciding with the profile of stabilized flow of a liquid of corresponding rheology in a circular tube.

2. Method of Solution. The problem formulated can be solved by a finite difference method, the basic concepts of which are described in [6]. Included in the calculation cycle of computing velocities and pressures are steps of determining effective viscosity and correction velocity potential values. The latter is an effective method of eliminating accumulation of approximation errors in satisfying the incompressibility condition [7].

Rheological equation (3) is taken in the form $\tau_{ij} = 2B\dot{e}_{ij}$. In dimensionless form the effective viscosity B is written in the following manner:

$$B = \frac{1}{A}\,(S^{1/n} + A^{1/m})^n. \tag{10}$$

In calculating flows with quasisolid cores or stagnant zones the use of Eq. (10) causes serious difficulties involving the singularity as $A \to 0$. In connection with this the effective viscosity is calculated with the modified equation

$$B = \frac{1}{(A+\varepsilon)}\,(S^{1/n} + (A+\varepsilon)^{1/m})^n. \tag{11}$$

This modification, which as $\varepsilon \to 0$ permits a limiting transition to the Shul'man model, makes possible indirect calculation of flows with the presence of quasisolid cores and stagnant zones. The value of the free parameter ε is chosen for stability of the numerical calculation with the smallest possible distortions of the flow character. Numerical experiment revealed that the value ε = 0.05-0.1 satisfies this condition for the entire range of rheological parameters studied.

3. Calculation Results. In the general case of a nonlinear-viscoplastic Shul'man medium the character of the flow which develops for slow ($Re \ll 1$) pouring and, consequently, the mass residue left in the vessel when the free surface reaches the pouring orifice is determined by the vessel geometry, initial fill height H_0, values of the nonlinearity parameters n, m, and the complexes S and W. The quantity S, the dimensionless nonlinear viscoplasticity parameter, is defined by the ratio of plastic and viscous forces acting in the flow. The complex W is the ratio of the generalized Reynolds number to the Froude number and characterizes the ratio of gravitational and viscous forces. It is necessary to clarify the character and degree of influence on the draining process of the rheological parameters S, n, and m. The fundamental dependences on W, which for a nonlinear-viscoplastic medium are qualitatively the same as for a Newtonian liquid, are presented in [3].

In studying outflow of nonlinear-viscous liquids (S = 0) for other conditions equal the defining parameter will be the rheological constant k = n/m. Values of mass residue referred to the mass at the initial time for draining from a conical reservoir with $\beta_0 = \pi/6$, $H_0 = 5.27$ as functions of the nonlinearity parameter for two values of the complex W are presented in Fig. 1 (curves 1, 2). With increase in k the value of the residue decreases for both values of W: The free surface maintains a horizontal form longer, the funnel forms at lower height, and a film of smaller thickness remains on the solid wall. Differences in

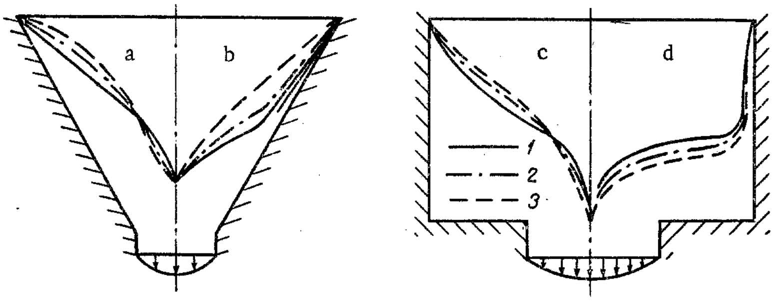

Fig. 2. Free surface forms: a, b) conical vessel; a) W = 0.126; b) W = 2; 1) k = 1.6; 2) 1; 3) 0.2; c, d) cylindrical tank; c) W = 0.8; d) W = 12.8; 1) k = 1.5; 2) 0.8; 3) 0.5.

the forms of the free surface at the time of completely formed funnels, leading to the indicated dependence of the mass residues, are shown in Fig. 2a, b. At the time of funnel formation and the almost instantaneously following breakthrough of air into the pouring orifice the vicinity of the axis of symmetry near the free surface is a zone of high deformation rates. As a consequence of this, for a pseudoplastic liquid the vicosity decreases toward the axis, which encourages funnel formation more rapid than for a Newtonian liquid. For a dilatant liquid the effective viscosity values increase toward the axis of symmetry, which leads to maintenance of a horizontal surface for a longer time than for a Newtonian liquid. It also follows from Fig. 2a, b that for the case of dominance of viscous forces over gravitational ones (W = 0.126) change in the parameter k leads to smaller differences in the mass residues as compared to the case where these forces are of the same order of magnitude (W = 2), since for decrease in k the increase in residues due to growth in thickness of the liquid film remaining on the walls is somewhat compensated by expansion of the valley region near the axis due to "liquefaction" of the pseudoplastic liquid in this zone of highest deformation rates.

The solid line of Fig. 2b shows a film constructed by the technique of [4] for k = 1.6, the form of which agrees quite well with the film form obtained in the present study.

We will consider the features which develop in draining of a nonlinear-viscous liquid from cylindrical vessels. In this case flow occurs with the presence of a turbulent zone near the corners of the vessel. In view of the low intensity of the turbulent motion these zones can be considered stagnant. The dependence of mass residue on the nonlinearity parameter is shown in Fig. 1 ($\beta = 2.5$, $H_0 = 3$, curves 3, 4). First, it can be concluded that for the given range of change in k and W the mass residue in the vessel at the moment when gas contacts the drain orifice depends only insignificantly on the nonlinearity parameter. Second, in the case where gravitational forces dominate over viscous ones (W = 12.8) there is some increase in residue with increase in k in contrast to W = 0.8 and the dependences constructed for a cone-shaped vessel. To explain these results we must commence from the character of the changes in the free surface form, which in turn are caused by the nonlinear dependence of viscosity on deformation rate. Free surface forms for various k are shown in Fig. 2c, d. For W = 0.8 the value of the nonlinearity parameter exerts the same effect on free surface form as in the case of a cone-shaped vessel. For sufficiently large W, in particular, at W = 12.8, the size of the stagnant zone at the vessel corners begins to affect the free surface form. This explains the surface locations for various k in Fig. 2d. Figure 3 shows the vertical size of the stagnant zone L as a function of the nonlinearity parameter at the initial time. The value of L was determined from the velocity fields obtained above. The triangle denotes the value of L obtained by solution of the problem of creeping flow of a Newtonian liquid in a channel with abrupt restriction [8]. The good agreement with the L value obtained in the presence of a free surface permits the conclusion that the effect of the latter on the flow is insignificnt in the vicinity of the sudden restriction. In addition, we will note that the dimensions of the stagnant zone undergo practically no change during the time of free surface motion until the latter reaches the drain orifice. Thus, the basic cause of increased mass residue with increase in the parameter k is the growth in size of the stagnant zone. Gravitational forces should here dominate over viscous ones (i.e., the number W should be quite high), in order that the outflow regime be realized with preservation of a horizontal free surface which reaches the vicinity of the stagnant zone, whereupon the effect of the latter appears.

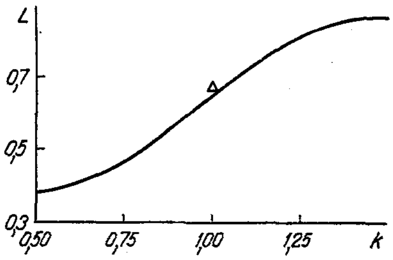

Fig. 3

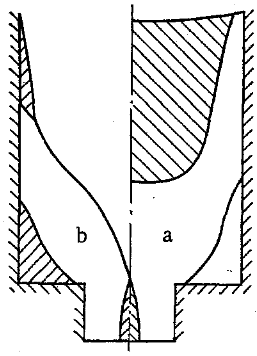

Fig. 4

Fig. 3. Stagnant zone size vs nonlinearity parameter for drainage from a cylindrical vessel ($\beta = 2.5$, $H_0 = 3$, $W = 0.8$).

Fig. 4. Flow structure: a) initial moment; b) time of gas contact with drain orifice.

The presence in the liquid rheological behavior of a yield point leads to significant changes in the drainage process. We will first consider the features found in numerical calculations for the case $n = m = 1$ (a Shvedov–Bingman plastic).

Residual masses as functions of the plasticity parameter S for cone-shaped and cylindrical vessels are shown in Fig. 1 (curves 5, 6). Increase in residue with growth in S initially occurs quite intensely ($S < 1$), after which the dependence becomes weaker. The corresponding changes in free surface are caused by an increase in effective viscosity in the flow region upon rise in viscoplastic properties. Like a decrease in the ratio of the Reynolds and Froude numbers, this leads to growth in the liquid layer which remains on the solid walls and an increase in the height of funnel formation. On the other hand, for a linear-viscoplastic medium, just like a pseudoplastic liquid, it is characteristic that the effective viscosity decreases with increase in deformation rate, which leads to increased slope of the free surface near the axis of symmetry and expansion of the depression region (funnel). This is the cause of the slowing in the rate of increase of residual mass with increase in plasticity parameter.

In the case of flow of a viscoplastic medium under consideration here the question of development and evolution of quasisolid cores and stagnant zones is of interest. As a condition for distinguishing quasisolid flow zones we may take

$$B \cdot A < S. \tag{12}$$

Equation (12) is a dimensionless analog of the following inequality

$$\sqrt{T_2} < \tau_0. \tag{13}$$

Inequality (13) defines the limits of quasisolid zones precisely only in the limiting case of a medium which obeys Shul'man's equation (3). Condition (13) was used in [9] to distinguish quasisolid cores in a study of convection of a viscoplastic liquid in closed regions. Numerical calculations performed with the Williamson model, which for $n = m = 1$ is equivalent to Eq. (11), have shown that for sufficiently small ε arbitrary distinction of such zones with Eq. (13) produces a good approximation of the flow structure of a Shvedov–Bingman liquid. In the present calculations it was also found that beginning at $\varepsilon \sim 0.1$ the boundaries of the zones distinguished undergo practically no change. For $\varepsilon < 0.01$ there is a significant increase in effective viscosity in the region of quasisolid flow, which is accompanied by an abrupt increase in the number of iterations required for convergence. The definition of quasisolid zones by a generalized rheological equation in the form of Eq. (11) is arbitrary in that we speak of smallness of deformation rates in those zones as compared to the viscous flow region, rather than total absence of deformation.

Evolution of quasisolid cores and stagnant zones was traced for the case of draining from a cylindrical vessel for the following parameter values: $\beta = 2.5$, $W = 3.2$, $H_0 = 6$, $n = m = 1$, $S = 1$. The flow structure is depicted in Fig. 4, where the cross-hatched areas denote quasisolid flow zones defined by Eq. (12). There is a core in the drain orifice which remains practically constant over time. A core forms in the vessel limited above by the free surface. Between the core and the solid walls viscous flow occurs. During the drainage process destruction of the core occurs, beginning in the vicinity of the axial point of the free surface and being completed by the time the free surface reaches the drain orifice. The stagnant zone at the corner of the vessel decreases insignificantly. After formation of a film on the solid wall yet another zone in which flow is absent forms, in the upper portion of the film. This zone increases with time.

In another variant of the calculation with the same data, but an initial filling height $H_0 = 3$, quasisolid flow zones are found in the drain orifice (central core) and the corners of the vessel (stagnant zone). No core is formed in the vessel itself in this case, so that a drainage regime with intense liquid motion in the vicinity of the axis and rapid funnel formation is realized. The free surface immediately takes on a convex form with a depression on the axis without any film formation on the solid wall.

Thus, there exist two characteristic regimes of drainage of viscoplastic media with different flow structures. These are distinguished by the ratio of the initial filling height to the critical height at which the funnel is formed (which for the case under consideration is approximately equal to three).

Nonlinear-viscoplastic liquids may manifest both pseudoplastic and dilatant properties depending on the ratio of the nonlinearity parameters n and m [5]. The effect of the parameters n and m on the drainage process was studied with a series of calculations for a cone-shaped vessel ($\beta_0 = \pi/6$, $H_0 = 5.27$, $W = 2$, $S = 0.55$, $n = 0.4$-1.6, $m = 1$-1.6) and a cylinder ($\beta = 2.5$, $H_0 = 3$, $W = 3.2$, $S = 1$, $n = 0.7$-2, $m = 1$-3). Analysis of the results obtained indicates that increase in m for n = const leads to changes in the free surface form at the moment of gas appearance in the drain orifice, equivalent to those observed upon increase in the pseudoplastic properties of a nonlinear-viscous liquid. In the case considered increase in the parameter m also leads to increase in the thickness of the liquid layer remaining near the walls, and expansion of the depression region near the axis of symmetry. Changes in the free surface form are accompanied by insignificant increase in the mass residue (by 3-4%). With increase in n (m = const) the changes in rheological behavior are of an opposite character, but for the range studied have practically no effect on the value of residual mass. Variation of the parameters n and m (n = m) over the range 0.7-2 also led to no significant differences, although there was a slight increase in residual mass for increase in n, m (~5%).

Thus, the rheological behavior of the liquid has a significant effect on the character of drainage from axisymmetric vessels only for significant deviations of the flow curve from Newton's law. The presence of a liquid yield point produces the greatest differences.

NOTATION

Π, stress tensor; ρ, density; g, gravitational force vector; V, velocity vector; T, deviator of stress tensor with components τ_{ij}; τ_0, yield point; μ, n, m, constants of the Shul'man model; $\dot{e}_{ij}$, deformation rate tensor components; $A = (2\dot{e}_{ij}\dot{e}_{ji})^{1/2}$, deformation rate intensity; p, pressure, I, unit tensor; S_1, free boundary; S, flow region boundary; $S_2 = S/S_1$; $V_0(x)$, specified velocity vector (on fixed solid wall $V_0(x) = 0$); n, s, unit vectors normal and tangent to free surface; f(x), function describing free surface; B, dimensionless effective viscosity; $S = \tau_0(R/\mu V)^{n/m}$, dimensionless nonlinear viscoplasticity parameter; R, drain orifice radius; V, modulus of mean flow velocity in drain orifice; ε, small parameter; $Re = \rho V^2(R/\mu V)^{n/m}$, generalized Reynolds number; $W = \rho g R(R/\mu V)^{n/m}$, ratio of Reynolds and Froude numbers; H_0, initial liquid level above drain orifice, referenced to R; β_0, conical vessel semiangle; β, ratio of cylindrical vessel diameter to drain orifice diameter; T_2, second invariant of stress tensor deviator.

LITERATURE CITED

1. I. I. Ievlev, Tr. Fiz. Tekh. Inst. Akad. Nauk Ukr. SSR, No. 1 [in Russian], Kharkov (1970), pp. 146-156.
2. R. A. Madsden, C. R. Easton, G. W. Burge, and I. Catton, J. Spacecr. Rockets, 7, No. 1, 89-91 (1970).
3. G. R. Shrager and V. A. Yakutenok, Problems of Viscous Liquid Dynamics [in Russian], Novosibirsk (1985), pp. 293-296.

4. Z. P. Shul'man and V. I. Baikov, Rheodynamics and Heat-Mass Exchange in Film Flows [in Russian], Minsk (1979).
5. Z. P. Shul'man, Convective Heat-Mass Transport of Rheologically Complex Liquids [in Russian], Moscow (1975).
6. I. M. Vasenin, A. P. Nefedov, and G. R. Shrager, Chisl. Met. Mekh. Sploshn. Sred., 16, No. 6, 28-43 (1985).
7. U. Pracht, Numerical Methods in Liquid Mechanics [in Russian], Moscow (1973), pp. 174-182.
8. D. Sigli and P. Monnet, Appl. Sci. Res., 7, No. 3, 215-232 (1982).
9. T. P. Lyubimova, Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza, No. 1, 3-8 (1977).

EXPERIMENTAL INVESTIGATION OF THE EFFECT OF LOW-FREQUENCY FLUCTUATIONS OF THE LIQUID FLOW RATE ON THE MINIMUM IRRIGATION DENSITY IN FILM FLOW

S. E. Shcheklein and V. I. Vel'kin

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It is shown that when low-frequency fluctuations (3-7 Hz) are imposed on a stream of liquid being fed to a channel, the minimum irrigation density in the film is reduced by a factor of 2-3 times compared with the case of nonfluctuating flow.

One of the fundamental technical characteristics of film flows and disperse-film flows is the existence of a minimum irrigation density [1]. This characteristic determines the conditions for the wetting of heat transfer surfaces by the liquid and has an important effect on heat transfer to the film, the value of the maximum wall temperature, and the critical heat flux corresponding to the occurrence of a heat transfer crisis of the second type.

A number of papers have dealt with experimental and theoretical investigations of the minimum irrigation density [2-6]. All the authors have mentioned the important effect on the value of the minimum irrigation density of the kinetic energy of the film flow and the surface tension energy at the interface. Different irrigation densities have been noted corresponding to the onset of breakup of the film, and irrigation densities at which dry spots which have formed are eliminated.

This difference, which is explained by the hysteresis of the wetting contact angle, amounts to 450-1200% or more, and depends on the temperature of the liquid at the inlet, the temperature difference at the inlet between the wall and the film, the roughness and cleanness of the material of the surface being wetted, and the construction of the distributing device.

For improving the wetting of dry surfaces by irrigation, recommendations have been made to use shaking [4] or vibration of the film equipment, or a brief considerable increase in the irrigation density [5].

It is obvious that all these measures are aimed at increasing the kinetic energy of the film flow to a value which exceeds the energy of the surface tension forces at the interface. At the same time, these methods of improving the wettability are very technologically inefficient, and often reduce the reliability of the equipment.

The present paper presents the results of an investigation of the possibility of increasing the kinetic energy of a wavy film flow by means of artificial perturbations imposed on the liquid stream by a bellows pulsator.

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